

New Limits on the Couplings of Light Pseudoscalars from Equivalence Principle Experiments

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(May 20, 1999)

Abstract

The exchange of light pseudoscalar quanta between fermions leads to long-range spin-dependent forces in order g^2 , where g is the pseudoscalar-fermion coupling constant. We demonstrate that laboratory bounds on the Yukawa couplings of pseudoscalars to nucleons can be significantly improved using results from recent equivalence principle experiments, which are sensitive to the spin-independent long-range forces that arise in order g^4 from two-pseudoscalar exchange.

04.90.+e, 14.80.Mz

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It is well known that the exchange of a light pseudoscalar quantum (ϕ) with mass m between two fermions (ψ) of mass M gives rise to a long-range spin-dependent fermion-fermion interaction. If we describe the fundamental coupling via the usual Lagrangian density

$$\mathcal{L}(x) = ig\bar{\psi}(x)\gamma_5\psi(x)\phi(x), \quad (1)$$

where g is the pseudoscalar coupling constant, then the spin-dependent potential between two identical spin-1/2 fermions is given by [1]

$$V^{(2)}(\vec{r}; \vec{\sigma}_1, \vec{\sigma}_2) = \frac{g^2}{16\pi M^2} \left\{ (\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) \left[\frac{m^2}{r} + \frac{3m}{r^2} + \frac{3}{r^3} \right] - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left[\frac{m}{r^2} + \frac{1}{r^3} \right] \right\} e^{-mr}. \quad (2)$$

Here $r = |\vec{r}| = |\vec{r}_1 - \vec{r}_2|$ is the distance between fermions 1 and 2, $(1/2)\vec{\sigma}_{1,2}$ are the fermion spins ($\hbar = c = 1$), and we have dropped a term proportional to $\delta^3(r)$. Our focus in this paper will be on the $m = 0$ limit [2] of Eq. (2), which characterizes the long-range interaction between fermions when $1/m$ is large compared to the size of the apparatus,

$$V^{(2)}(\vec{r}; \vec{\sigma}_1, \vec{\sigma}_2) \xrightarrow{m=0} \frac{g^2}{16\pi M^2} \frac{S_{12}}{r^3}, \quad (3a)$$

$$S_{12} \equiv 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2). \quad (3b)$$

Limits on $g^2/4\pi$ derived from recent spin-dependent experiments are summarized by Ritter, *et al.* [3]. Although these limits appear at first to be quite restrictive, they are not nearly as stringent as the limits implied by recent spin-independent tests of the equivalence principle, which also probe for the presence of new long-range forces. For example, if the coupling of a new long-range vector field A_μ to fermions is described by the Lagrangian

$$\mathcal{L} = if \bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(x), \quad (4)$$

then typical limits on $f^2/4\pi$ over laboratory distance scales are $f^2/4\pi \lesssim 10^{-46}$ [4,5] compared to $g_e^2/4\pi \lesssim 10^{-16}$, where g_e is the pseudoscalar coupling to electrons [3]. Among the reasons

for the differing sensitivities of spin-dependent and spin-independent experiments are [5]: 1) The strength of the spin-dependent coupling in Eq. (3a) is suppressed relative to that for the spin-independent coupling by a factor of order $1/(MR)^2$, where R is the characteristic size of the experimental apparatus. If M denotes the electron mass and $R = 1$ m, then $1/(MR)^2 \simeq 1.5 \times 10^{-25}$. 2) Test masses which have a net electron-spin polarization can also interact electromagnetically. Since the electromagnetic background is many orders of magnitude larger than the effects expected from a putative new force, special materials (such as $\text{Dy}_6\text{Fe}_{23}$) and methods must be used which limit the sizes of the samples that can be studied. 3) Furthermore, even in these special materials, only a small fraction of the test masses actually contributes, since the net polarization is only 0.4 electrons per $\text{Dy}_5\text{Fe}_{23}$ molecule [6]. 4) The spin-dependent couplings of light pseudoscalars to nucleons are further suppressed by the dilution of the electron polarization as it is transferred to the nucleons.

The disparity in the limits set on g^2 and f^2 , by spin-dependent and spin-independent experiments respectively, raises the question of whether interesting limits on g^2 can also be inferred from spin-independent searches for macroscopic forces. The exchange of two pseudoscalars, as shown in Fig. 1, gives rise to a spin-independent potential $V^{(4)}(r)$ in order g^4 which has been calculated by a number of authors [7,8]. In the limit $m \rightarrow 0$, $V^{(4)}(r)$ is given by

$$V^{(4)}(r) = -\frac{g^4}{64\pi^3 M^2} \frac{1}{r^3} \equiv g^4 f(r). \quad (5)$$

Interestingly, $V^{(4)}$ and $V^{(2)}$ have the same functional dependence on M and r in the $m = 0$ limit, and the ratio of their strengths (per pair of interacting particles) is

$$\frac{|V^{(2)}(r; \vec{\sigma}_1, \vec{\sigma}_2)|}{|V^{(4)}(r)|} = \frac{4\pi^2 |\langle S_{12} \rangle|}{g^2}, \quad (6)$$

where $\langle S_{12} \rangle$ is determined by averaging over the polarizations of samples 1 and 2. We see from Eq. (6) that although $V^{(4)}$ is suppressed relative to $V^{(2)}$ by the factor $g^2/4\pi^2$, $V^{(2)}$ is suppressed relative to $V^{(4)}$ by the factor $\langle S_{12} \rangle$. Moreover, $V^{(2)}$ is further suppressed relative to $V^{(4)}$ by virtue of the fact that there are fewer contributions to $\sum V_{ij}^{(2)}$ than to $\sum V_{ij}^{(4)}$, since the source masses are necessarily smaller in the spin-dependent experiments.

As we show in the ensuing discussion, the net effect of the various suppression factors in Eq. (6) is that the most stringent laboratory limits on Yukawa couplings of pseudoscalars to protons, neutrons, (and ultimately quarks) arise from spin-independent equivalence principle experiments which constrain $V^{(4)}$, rather than from spin-dependent experiments which are sensitive to $V^{(2)}$. Since the couplings of axions to fermions involve derivatives, the resulting 2-axion potential varies as $1/r^5$ rather than as $1/r^3$, as has been noted by Ferrer and Grifols [8]. Hence, the numerical results of the present paper do not apply to axions directly, although the present formalism can be taken over for axions with appropriate modifications.

Consider the interaction between two objects 1 and 2 containing N_1 (Z_1) neutrons (protons), and N_2 (Z_2) neutrons (protons), respectively. The total energy W is obtained by summing the pairwise interactions arising from Eq. (5) after replacing the generic coupling constant g^4 by g_n^4 , g_p^4 , or $g_n^2 g_p^2$ for n-n, p-p, and n-p interactions, respectively. Here g_n (g_p) denotes the pseudoscalar coupling constant appearing in Eq. (1) when ψ is a neutron (proton). From Eq. (5) W can be expressed in the form

$$W = [g_p^4 Z_1 Z_2 + g_n^4 N_1 N_2 + g_n^2 g_p^2 (Z_1 N_2 + Z_2 N_1)] \langle f(r) \rangle, \quad (7)$$

where $\langle f(r) \rangle$ is obtained from Eq. (5) by integrating over the mass distributions of the two objects.

In a typical equivalence principle experiment object 1 is an extended source toward which the relative accelerations of samples 2 and 2' (with masses M_2 and $M_{2'}$) are being measured. If the dimensions of the test masses are small compared to the size of the source, the force $\vec{F}(\vec{r})$ exerted by the source on test mass 2 (located at \vec{r}) can be written in the form

$$\vec{F}(\vec{r}) = [g_p^4 Z_1 Z_2 + g_n^4 N_1 N_2 + g_n^2 g_p^2 (Z_1 N_2 + Z_2 N_1)] \vec{\mathcal{F}}(\vec{r}), \quad (8a)$$

$$\vec{\mathcal{F}}(\vec{r}) = \left(\frac{-3}{64\pi^3 M^2 \mathcal{V}_1} \right) \int d^3 r'_1 \frac{(\vec{r} - \vec{r}'_1)}{|\vec{r} - \vec{r}'_1|^5}, \quad (8b)$$

where \mathcal{V}_1 is the volume of the source. It follows from Eq. (8a) that the experimentally measured acceleration difference $\Delta \vec{a}_{2-2'} \equiv \vec{a}_2 - \vec{a}_{2'}$ is given by

$$\Delta \vec{a}_{2-2'} = \vec{\mathcal{F}}(\vec{r}) \left(\frac{M_1}{m_H^2} \right) \left[g_p^2 \left(\frac{Z_1}{\mu_1} \right) + g_n^2 \left(\frac{N_1}{\mu_1} \right) \right] \left[g_p^2 \Delta \left(\frac{Z}{\mu} \right)_{2-2'} + g_n^2 \Delta \left(\frac{N}{\mu} \right)_{2-2'} \right], \quad (9)$$

where M_1 is the source mass, $\Delta(Z/\mu)_{2-2'} = Z_2/\mu_2 - Z_{2'}/\mu_{2'}$, etc., $\mu_i = M_i/m_H$, and $m_H = m({}_1\text{H}^1)$ [5]. Except for g_p^2 and g_n^2 , the right-hand side of Eq. (9) is known, and hence an experimental determination of $\Delta \vec{a}_{2-2'}$ leads to a constraint on g_p^2 and g_n^2 .

Examination of Eq. (9) leads to the observation that there are two classes of constraints on g_p^2 and g_n^2 , depending on the relative signs of $\Delta(Z/\mu)_{2-2'}$ and $\Delta(N/\mu)_{2-2'}$. Since g_p^2 , g_n^2 , N_1 , and Z_1 are all inherently positive, the right-hand side of Eq. (9) cannot vanish if $\Delta(Z/\mu)_{2-2'}$ and $\Delta(N/\mu)_{2-2'}$ have the same sign, unless g_p^2 and g_n^2 themselves do. It follows that in this circumstance an experimental bound on $\Delta \vec{a}_{2-2'}$ leads to an absolute upper bound on either g_p^2 or g_n^2 . We refer to such constraints as “elliptical”, since Eq. (9) produces ellipses in the x - y plane defined by $x = g_p^2$ and $y = g_n^2$. By contrast, if $\Delta(Z/\mu)_{2-2'}$ and $\Delta(N/\mu)_{2-2'}$ have opposite signs, the right-hand side of Eq. (9) can vanish whenever g_p^2 and g_n^2 satisfy

$$\frac{g_p^2}{g_n^2} = -\frac{\Delta(N/\mu)_{2-2'}}{\Delta(Z/\mu)_{2-2'}}, \quad (10)$$

and hence g_p^2 and g_n^2 can be arbitrarily large and still be compatible with any experimental bound on $\Delta \vec{a}_{2-2'}$. We term such constraints “hyperbolic”, since in this case Eq. (9) leads to hyperbolas in the x - y plane. The asymptotes of these hyperbolas in the (physical) first quadrant lie near the line $y = x$, which represents the locus of points satisfying Eq. (10) [9].

It is instructive to contrast the constraints arising from $V^{(4)}$ in Eq. (5) with those arising in second order from the exchange of a scalar or vector field, as in the usual “fifth force” scenario [5]. The expression for $\Delta \vec{a}_{2-2'}$ in this case has the same general form as in Eq. (9) except that $g_{p,n}^2 \rightarrow g_{p,n}$. Since g_p and g_n can each be positive or negative, no choice of samples 2 and 2' can ensure that the coefficient of $\vec{\mathcal{F}}(\vec{r})$ will have a unique sign, and hence there are no elliptical constraints in the conventional “fifth force” case. Note that $\Delta \vec{a}_{2-2'}$ can vanish not only when the analog of Eq. (10) holds for the test masses, but also when the source strength vanishes as happens when $(g_p Z_1 + g_n N_1) = 0$ [10]. It follows from this discussion that the novel feature of $V^{(4)}$ is that it gives rise to elliptical constraints, and

hence to absolute bounds on g_p^2 and g_n^2 , for appropriate choices of 2 and 2'. To determine which pairs of elements would produce elliptical constraints, we have evaluated $\Delta(Z/\mu)$ and $\Delta(N/\mu)$ for the 4,186 pairs that can be formed from the first 92 elements, and found 7 possible pairs: He-N, He-O, N-O, S-Ca, Br-Mo, Li-Ru, and Pt-Rn. Among these, Li-Ru is the most obvious choice, where the Li sample could be gold-plated to prevent oxidation. Other choices involving compounds are also possible, as we discuss in greater detail elsewhere [9].

As we now demonstrate, if the preceding formalism is combined with the recent results of Gundlach, *et al.* [4], the laboratory limits on g_p^2 and g_n^2 can be significantly improved. This experiment compared the accelerations of test bodies composed of Cu and a Pb alloy toward a 2620 kg depleted U source, and they found for the acceleration difference

$$\Delta \vec{a}_{2-2'} = \vec{a}_{\text{Cu}} - \vec{a}_{\text{Pb}} = \hat{r}(-0.7 \pm 5.7) \times 10^{-13} \text{ cm/s}^2, \quad (11)$$

where \hat{r} is a unit vector in the direction of the field $\vec{\mathcal{F}}$ produced by the source. Since the U source was positioned close to the test masses, this experiment can be used to set limits on short-range interactions of the form $V(r) = \Lambda_N(r_0/r)^{N-1}(\hbar c/r)$, with $N = 3$ corresponding to $V^{(4)}$ in Eq. (5). Combining Eq. (9) with the bound from Ref. [4], $\Lambda_3 < 6 \times 10^{-16}$, leads to the constraint

$$\frac{|\Delta \vec{a}_{2-2'}|}{(1 \text{ cm/s}^2)} = (9.6g_p^2 + 15.3g_n^2)|g_p^2\Delta(Z/\mu)_{2-2'} + g_n^2\Delta(N/\mu)_{2-2'}|, \quad (12)$$

which applies to any test masses 2 and 2' in Ref. [4]. For the actual samples used, 2 = Cu and 2' = Pb alloy, $\Delta(Z/\mu)_{2-2'} = 0.05925$, $\Delta(N/\mu)_{2-2'} = -0.05830$, and the slope of the asymptote for the hyperbolic constraint implied by Eq. (12) is $0.05925/0.05830 = 1.016$. Inserting these results for 2 and 2' into Eq. (12) along with the 1σ bound in Eq. (11), $|\Delta \vec{a}_{2-2'}| < 6.4 \times 10^{-13} \text{ cm/s}^2$, leads to the final result,

$$(9.6g_p^2 + 15.3g_n^2)|0.05925g_p^2 - 0.05830g_n^2| \lesssim 6.4 \times 10^{-13}. \quad (13)$$

A plot of the hyperbolic constraint in Eq. (13) is shown in Fig. 2 along with an illustrative elliptical constraint curve obtained from Eq. (12) by substituting 2 = Li and 2' = Ru. As

can be seen from this figure and Eqs. (12) and (13), separate bounds on g_p^2 and g_n^2 can be inferred by repeating the experiment of Gundlach, *et al.*, with various combinations of appropriately chosen test masses. Even though the relevant experiments have not yet been performed, one can nonetheless obtain useful bounds on g_p^2 and g_n^2 separately by considering special cases of Eq. (13). For example, for a light pseudoscalar which couples universally to baryon number we have $g_p^2 = g_n^2$, and hence from Eq. (13)

$$g_p^2/4\pi \lesssim 4 \times 10^{-7}. \quad (14)$$

The result in Eq. (14) represents an improvement by more than two orders of magnitude on the bound inferred by Ramsey [11,12], $g_p^2/4\pi \lesssim 5 \times 10^{-5}$. This is the only other direct laboratory limit on g_p^2 , which was obtained by comparing theory and experiment for the energies of low-lying vibrational and rotational states in molecular H_2 . Two other interesting bounds can be inferred from Eq. (13) in the limiting cases $g_p^2 \gg g_n^2$ and $g_n^2 \gg g_p^2$. These are

$$g_p^2/4\pi \lesssim 9 \times 10^{-8}, \quad (g_p^2 \gg g_n^2); \quad g_n^2/4\pi \lesssim 7 \times 10^{-8}, \quad (g_n^2 \gg g_p^2). \quad (15)$$

In contrast to the case for g_p^2 , there are no direct laboratory limits on g_n^2 , apart from those arising from Eq. (13). However, one can attempt to infer a crude indirect bound on g_n^2 by following an argument due to Daniels and Ni (DN) [13]. Consider, for example, the experiment of Ritter, *et al.* [6], which uses test samples of $\text{Dy}_6\text{Fe}_{23}$ containing polarized electrons to measure g_e^2 . As noted by DN, the hyperfine interaction of the electrons in Dy aligns the Dy nuclei and similarly, but less significantly, for Fe. DN estimate this polarization (at room temperature) to be $P_X \simeq 3.4 \times 10^{-5}$ ($X = \text{Dy}$), which compares to $P_e \simeq 0.4$ for the electrons themselves. Hence, although the Dy nuclei have a non-zero induced polarization, this polarization is quite small. It follows that the sensitivity of the experiment of Ritter, *et al.* [6] to g_X^2 is smaller than its sensitivity to g_e^2 by a factor $P_X^2/P_e^2 = 7 \times 10^{-9}$, due to the differences in $\langle S_{12} \rangle$ for electrons and nuclei. To infer a bound on g_n^2 the Dy polarization must be related to that of the neutron. If we assume, for example, that the polarization of the Dy nucleus is carried by a single odd neutron outside a symmetric core, then we can identify

the neutron polarization with that of the Dy nucleus. Combining the preceding arguments we are led to the crude estimate,

$$g_n^2/4\pi \lesssim (P_X^2/P_e^2)^{-1}(g_e^2/4\pi) \simeq 8 \times 10^{-6}, \quad (16)$$

where we have used $g_e^2/4\pi \lesssim 6 \times 10^{-14}$ from Ritter, *et al.* [6]. Note that although the limits on g_e^2 from other experiments such as Ref. [14,15] are more restrictive, the configuration of these experiments renders the preceding arguments inapplicable [9]. In the experiment of Chiu and Ni [14], for example, the polarization of an initially unpolarized TbF_3 sample was measured in the presence of a rotating polarized $\text{Dy}_6\text{Fe}_{23}$ source. Since the TbF_3 sample was shielded against conventional magnetic fields by superconducting Nb, any polarization of the electrons would arise solely from the putative long-range spin-spin interaction, which is presumably a small effect. The alignment of the nuclear spins via the hyperfine interaction would be smaller still, and hence no useful limit on couplings to nucleons emerges from such an experiment.

The laboratory constraints on pseudoscalar couplings derived in this paper are model independent, but do not apply to axions which are derivative-coupled [8]. Although the present formalism can be adapted to infer limits on axion couplings using the $1/r^5$ potential arising from 2-axion exchange, the best existing limits on light axions still come from stellar cooling [16,17]. In addition, astrophysical arguments also yield tighter bounds on Yukawa (i.e., non-derivative) couplings of pseudoscalars to nucleons. For example, energy loss arguments from the SN 1987A supernova typically give $g^2/4\pi \lesssim 10^{-21}$ [16,18].

In summary, we have shown that the most stringent laboratory limits on the Yukawa couplings of light pseudoscalars to nucleons (and ultimately to quarks) derive from the $O(g^4)$ contributions in Fig. 1 to equivalence principle experiments. These limits can be further improved by reconfiguring existing experiments to make them more sensitive to a short-range $1/r^4$ force, and by using appropriate materials such as Li and Ru. Furthermore, by suitably adapting space-based experiments such as STEP [19] even more significant improvements in sensitivity could be realized in the foreseeable future.

The authors wish to thank Jens Gundlach, Wei-Tou Ni, Frank Rickey, and David Wineland for helpful communications. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC 02-76ER01428.

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FIGURES

FIG. 1. Contributions to the spin-independent long-range interaction of fermions a and b arising from two-pseudoscalar-exchange. The solid lines are fermions and the dashed lines denote the pseudoscalars.

FIG. 2. Constraints on g_p^2 and g_n^2 arising from two-pseudoscalar-exchange. The region shaded in dark gray exhibits the hyperbolic constraint implied by the experiment of Gundlach, *et al.*, Ref. [4]. The light gray region illustrates the hypothetical elliptical constraint that would emerge from Gundlach, *et al.*, had they used Li and Ru as the test masses. The overlap region is shown in black.